this maximum width, the difference in the radii of curvature of the two cavity walls at this point, and the curvature of the mean line at this point. With this concept it is possible to maximize the acoustic frequencies allowable in the chamber, to do this with a smooth, somewhat arbitrary shape, and to provide a degree of dispersion, i.e., the amplitude decays with distance from the maximum-width point.

The maximization of frequency is known to have beneficial effects concerning combustion instability. The arbitrariness of shape away from the maximum-width point allows freedom concerning manufacturing and weight problems. The dispersive mechanisms are important in determining stability characteristics of an engine. However, even given the assumption concerning the baffle effect, it should be remembered that $\epsilon \ll 1$ is required, and the meaning of "very much less than" can only be determined experimentally. The theory is at best an asymptotic representation of the true state of affairs and should not be applied blindly. Since the allowable number of baffles in an engine is necessarily limited (the magnitude of ϵ is limited), care must be taken in interpreting experimental results.

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Pressure Orifice Shape Effect in Rarefied Flow with Heat Transfer

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Nomenclature

A = area
b = oblong orifice streamwise dimension
d = circular orifice diameter
d_{eq} = equivalent oblong orifice diameter
M_∞ = freestream Mach number
$\dot{q}$ = heat-transfer rate
p_i = pressure indicated or measured in orifice cavity
p_{i0} = pressure on surface outside the orifice
Re_∞ = freestream Reynolds number
T_0 = total temperature
T_w = wall temperature
λ_i = mean free path based on measured pressure p_i and wall temperature

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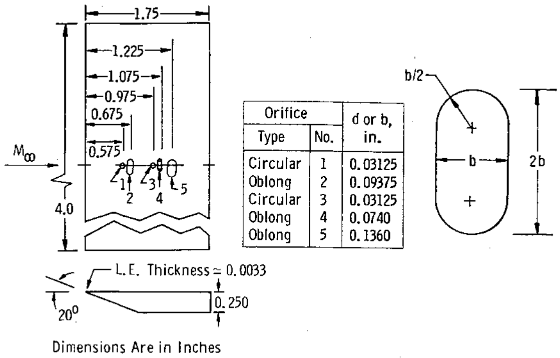

Orifice Type	No.	d or b, in.
Circular	1	0.03125
Oblong	2	0.09375
Circular	3	0.03125
Oblong	4	0.0740
Oblong	5	0.1360

Fig. 1 Flat plate model and pressure orifices.

The relation between the pressure in an orifice cavity and the true pressure on the surface of a cooled or heated body in rarefied, but not necessarily free-molecular, flow has been recently reported by Potter, Kinslow, and Boylan.[1] It was shown that, because of a form of nonequilibrium arising from unequal speed distributions that may exist between incoming and outgoing molecules in the orifice entrance region, the pressure sensed in the orifice cavity may be considerably in error.

Pressure data from impact-pressure probes and viscous interaction tests on flat plates and cones were used to illustrate these phenomena.[1] In addition, a semiempirical correction was derived by combining theoretical analysis for the limit of $d/\lambda_i \approx 0$ with experimental data obtained for cases where $d/\lambda_i \gg 0$.

All of the data described in Ref. 1 were obtained with circular-shaped orifices. A preliminary experimental investigation has been conducted to determine the effect of an oblong-shaped orifice on the corrections and thereby obtain a correction for pressures measured with oblong orifices.

The experiment was conducted in the von Karman Gas Dynamics Facility low-density wind tunnel (*L*). The model used was a sharp, flat plate, constructed of brass and water-cooled. The orifice locations and shapes are given in Fig. 1. Three different-sized oblong orifices and two circular orifices were used. The circular orifices were included to insure compatibility with a previous experiment. Based on thermocouple measurements, the plate surface temperature was about 300°K. The wind tunnel was operated at a freestream Mach number of 10.15 and Reynolds number of 388/in. with a total temperature of about 3100°K.

The pressure over the plate clearly showed an influence of orifice size as predicted by Potter, Kinslow, and Boylan.[1] Data from the present experiment agreed well with previous measurements and, in addition, showed how the oblong orifice shape affected the results. The results are presented in Fig. 2.

Shown are data for circular and oblong orifices. The dimension d refers to the diameter of the circular orifice, and b refers to the streamwise dimension of the oblong orifice. The measured pressure p_i is ratioed to the true surface pres-

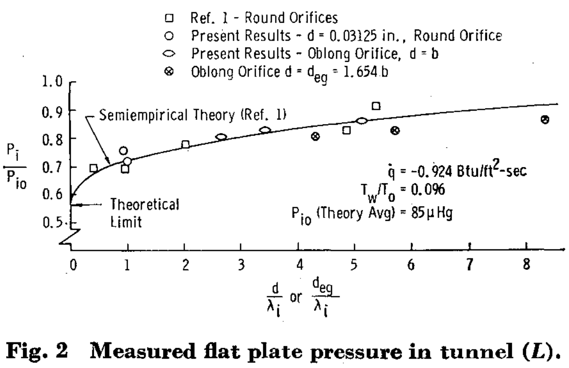

Fig. 2 Measured flat plate pressure in tunnel (*L*).

sure p_{i0} that was determined from prior experiments with circular orifices and the theory of Potter et al. The data from the circular orifices agree with the previous data from circular orifices, with the semiempirical-theoretical line splitting the small differences in measurements.

Also shown are the data from the oblong orifices, using an equivalent diameter d_{eq} to see if better correlation resulted. The equivalent diameter was determined by equating the orifice area to the area of a circle with an equivalent diameter, i.e.,

$$A_{\text{orifice}} = (\pi/4)\, d_{eq}^2$$

Although the results are not entirely conclusive, this method of handling the data does not appear to be as good as simply using the streamwise dimension b as the effective diameter.

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Unsteady Laminar Jet Flame at Large Frequencies of Oscillation

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Nomenclature

a = characteristic length in problem, jet width
c = speed of sound
D_{12} = binary diffusion coefficient
F = steady-state stream function
H, S, V, Z, U, W = high frequency representations of solution for P, σ, and $\mathcal{Y}_K$
h, s, v, z, u, w = high frequency expansion functions for H, S, V, Z, U, and W
i = complex number, $(-1)^{1/2}$
j = stoichiometric mass ratio of fuel to oxidizer
m = total mass flux per unit flame length/2π in r direction
M = Mach number
$\mathfrak{M}$ = space-dependent part of m perturbation
p = pressure
P = space-dependent part of ψ perturbation
q = stoichiometric heat of reaction nondimensionalized by $\bar{c}_p{}^*\bar{T}_\infty{}^*$
r = radial coordinate
R_f = perturbation magnitude of flame movement
Re = Reynolds number
s = axial coordinate
t = time
T = temperature
$u, (v)$ = velocity of center of mass of a fluid element in the s direction (r direction)
$x = s/Re$ = axial coordinate
Y_K = mass fraction of Kth species
$\mathcal{Y}_K$ = space-dependent part of Y_K perturbation
α = $1/(i\omega)^{1/2}$
β = high frequency variable
γ = ratio of specific heats
δ = ± 1
ϵ = perturbation parameter
μ = viscosity
$\hat{\mu}$ = boundary-layer variable
ξ = axial variable
ρ = density
σ = space-dependent part of T perturbation
$\tau = t/Re$ = time
ψ = stream function
ω = frequency

Subscripts

c = core centerline value
f = quantity evaluated at flame
F = fuel or fuel side of flame
o = oxidizer or oxidizer side of flame
∞ = freestream values

Superscripts

$\bar{\ }$ = steady-state quantity
$*$ = dimensional quantity
(i) = ith term in expansion in a small parameter

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INFORMATION concerning time-dependent combustion processes is very meager but greatly needed. Of several types of unsteadiness or time dependence, the most interesting from the point of view of unstable processes is the periodic phenomenon caused by the action of a periodic sound wave on an otherwise steady-state burning configuration. It appears that an interesting system for study is the laminar jet flame or overventilated diffusion flame. This system may have application to the wake of a burning droplet or the coaxial jet injector used in rocket engines. The occurrence of sound waves acting on such configurations is well known through the knowledge gained on the phenomenon of combustion instability.

High frequency of an imposed sound wave will be considered. "High" is meant in the sense that the cycle time is short compared to a particle transit time of the flame length. This is equivalent to the statement that the cycle time is short compared to a diffusion time transverse to the mixing region.

A method for generating an asymptotic solution valid in the entire flow field is developed. This method for the compressible, reacting, viscous jet may be specialized for simpler cases; as such, the method is general and contributes to unsteady boundary-layer theory. The primary interest in this note from the standpoint of application is to extract information concerning the behavior of the burning rate under the action of the sound wave. A complication in unsteady boundary-layer theory, therefore, enters: that of a moving boundary, the flame. The procedure for treating this is indicated. It should be made very clear that the interest here is in a forced oscillation, at the space and timewise frequency of a sound wave, which is compatible with the boundary-layer equations. This has no bearing on other possible disturbance types, e.g., an intrinsic flow instability consisting of disturbances traveling with the velocity of the fluid rather than at the speed of sound. It is well known that the type treated is most important in determining combustion energy feedback to an acoustic wave. The frequencies of intrinsic oscillations are, in general, not compatible with acoustic frequencies, and no organized interaction occurs, at least for small amplitudes.

Consider the laminar jet flame composed of a fuel rich gas ejected from a pipe into a flowing freestream containing an oxidizer as in Fig. 1. The jet velocity profile is characterized by a velocity $u_c{}^*$ and the freestream by a velocity $u_\infty{}^*$. It is assumed that $|u_\infty{}^* - u_c{}^*|$ is sufficiently large that $Re = \rho_\infty{}^*|u_\infty{}^* - u_c{}^*|a^*/\mu_\infty{}^* \gg 1$. At the same time, the respective Mach numbers $M_\infty = u_\infty{}^*/c_\infty{}^*$ and $M_c = u_c{}^*/c_c{}^*$ are low, such that their squares are negligible compared to unity. The jet is characterized by Y_{F_c} and $T_c{}^*$ and the freestream by Y_{o_∞} and $T_\infty{}^*$. Flame ignition is assumed instantaneous such that the flame is initiated at the lip of the jet. The fuel diffuses toward the flame from the core, and the oxidizer